AZPH-TH/96-11
LBL-38582

Dilepton and Photon Productions from a Coherent Pion Oscillation

Zheng Huang* and Xin-Nian Wang†

*Department of Physics, University of Arizona, Tucson, AZ 85721, USA

†Nuclear Science Division, MS 70A-3307

Lawrence Berkeley National Laboratory, Berkeley, CA 94720, USA

(April 10, 1996)

Abstract

Since the electromagnetic current for a pion system coincides with the third component of the isovector current, the isospin angular oscillation of a coherent field can be a significant source for the electromagnetic emissions. We study the characteristic dilepton and photon emissions from the classical pion field oscillation in the QCD vacuum. The general analytical solution obtained in the nonlinear σ model is used to calculate the electromagnetic current density, which exhibits a light-front singularity and decreases rapidly as inverse square of the proper time due to a longitudinal expansion. The momentum and invariant mass spectra of the direct photon and dilepton are found to be a sensitive probe of the space-time evolution of the chiral condensate field.

---

*huang@physics.arizona.edu

†xnwang@nsdssd.lbl.gov

Recently, there has been some interest in the formation of a disoriented chiral condensate (DCC) in high energy hadronic and heavy-ion collisions [1]. Preliminary studies have suggested that there may be a coherent pion field oscillation following a nonequilibrium second order phase transition which may lead to characteristic soft pion production [2,3]. According to a "Baked Alaska" scenario suggested by Bjorken, Kowalski and Taylor [1], the collision debris in a high energy central event form a "hot" source on the light front and move outward while emitting quasi-goldstone bosons in the cool interior of light cone. These pions are not assumed to reach a thermal equilibrium, rather, they behave collectively and a coherent field description is more relevant. The space-time evolution of the condensate field is mainly determined by the classical equation of motion for a low energy effective theory such as a linear or nonlinear σ model. Eventually, the "disoriented vacuum" relaxes back to normal vacuum by emitting physical pions. It is, however, difficult to obtain the information on its space-time evolution from the final hadron spectrum.

In contrast, the electromagnetic signature is known to be an ideal probe of a dense hadronic matter [4] owing to the fact that it escapes the strong interaction region once produced without further final state interactions and thus carries the information on the early dynamical evolution. In this Letter, we shall study the dilepton and direct photon production from the classical pion field in the context of a disoriented chiral condensate (DCC) or most generally a nonequilibrium pion cloud. We shall develop a general formalism for the dilepton and photon emissions in the presence of a classical electromagnetic current. As an example, we calculate the dilepton and photon spectra in an analytical solvable model where a general class of solutions are obtained by Blaizot and Krzywicki [5] and by Huang and Suzuki [6]. Although the model is a simplified version of a more realistic situation, it captures the essential features of soft pion production such as the light cone singularity, the boost invariance and the longitudinal expansion. The electromagnetic spectra are found to be sensitive to the dynamical evolution of the classical pion field. Since the electromagnetic current coincides with the third component of the isovector current, the isospin angular oscillation of the condensate field can be a significant source for the electromagnetic emission

in the low mass (transverse momentum) region.

The pionic part of electromagnetic interactions is introduced by replacing (we neglect the anomalous electromagnetic coupling) $\partial_\mu \boldsymbol{\pi}$ by $\partial_\mu \boldsymbol{\pi} + \frac{ie}{\sqrt{2}}[\boldsymbol{\pi}, Q]\mathcal{A}_\mu$ in the kinetic part of lagrangian

$$\mathcal{L}^{\rm em}(x) = ie[\pi^-(x)\partial_\mu \pi^+(x) - \pi^+(x)\partial_\mu \pi^-(x)]\mathcal{A}^\mu(x) = -eJ^{\rm cl}_\mu(x)\mathcal{A}^\mu(x) \ , \tag{1}$$

where $\mathcal{A}_\mu$ is the photon field and $J^{\rm cl}_\mu$ is the third component of the classical isospin current $J^{\rm cl}_\mu(x) = (\boldsymbol{\pi}(x) \times \partial_\mu \boldsymbol{\pi}(x))_3$. To $O(e^2)$, the matrix element for the dilepton emission from the classical field [7–10] is

$$\mathcal{M} = \langle \ell^+\ell^-|S^{(2)}|0\rangle = e^2 \bar{u}(p_1)\gamma_\nu \upsilon(p_2) D^{\nu\mu}(p_1+p_2) J^{\rm cl}_\mu(p_1+p_2) \ , \tag{2}$$

where $J^{\rm cl}_\mu(p_1+p_2)$ is the Fourier transformation of $J^{\rm cl}_\mu(x)$, u and υ are the Dirac spinors for the electron and positron, $D^{\nu\mu}$ is the photon propagator. The number of dileptons produced per final state phase space is

$$\begin{aligned} dN_{\ell^+\ell^-} &= \sum_{\rm spin} |\mathcal{M}|^2 \frac{d^3p_1}{(2\pi)^3 2E_1} \frac{d^3p_2}{(2\pi)^3 2E_2} \\ &= \int d^4q \delta^4(q-p_1-p_2) \sum_{\rm spin} |\mathcal{M}|^2 \frac{d^3p_1}{(2\pi)^3 2E_1} \frac{d^3p_2}{(2\pi)^3 2E_2} \ . \end{aligned} \tag{3}$$

The dilepton differential distribution with respect to the lepton pair 4-momentum q is then

$$\frac{dN_{\ell^+\ell^-}}{d^4q} = \frac{\alpha^2}{6\pi^3}\frac{B}{q^4}[q^\mu q^\nu - q^2 g^{\mu\nu}] J^{\rm cl}_\mu(q) J^{\rm cl*}_\nu(q) \ , \tag{4}$$

where $B = [1 + 2m_\ell^2/q^2][1 - 4m_\ell^2/q^2]^{1/2}$ and m_ℓ is the lepton mass. Similarly, the direct photon momentum distribution is calculated

$$\frac{dN_\gamma}{d^3\boldsymbol{q}/\omega_q} = -\frac{\alpha}{(2\pi)^2} J^{\rm cl\mu}(q) J^{\rm cl*}_\mu(q) \ , \tag{5}$$

where q stands for the photon 4-momentum. Moreover, splitting the current into parts parallel and orthogonal to q_μ: $J^{\rm cl}_\mu(q) = q_\mu J^{\rm para}(q) + J^{\rm tr}_\mu(q)$, one finds that only the orthogonal part contributes to the emissions. The dependence of the electromagnetic spectra on the space-time history is entirely contained in the Fourier transformation of the isospin current.

Although some more realistic numerical simulations on the classical pion field dynamics exist [3] and ideally one would like to perform the Fourier transformation on these solutions, we do not attempt to do it in this Letter. Instead, we examine some simple scaling solutions in the 1+1 boost invariant model which are first obtained by Blaizot and Krzywicki [5]. Since the calculation is fully analytical, we hope to understand some qualitative features of dilepton production from the classical field.

The lagrangian for the nonlinear σ model is

$$\mathcal{L} = \frac{f_\pi^2}{4}\mathrm{tr}\Big[\partial_\mu \Sigma^\dagger(x)\partial^\mu\Sigma(x)\Big] + \frac{m_\pi^2 f_\pi^2}{4}\mathrm{tr}\Big[\Sigma^\dagger(x)+\Sigma(x)\Big], \tag{6}$$

where one defines the pion field by $\sigma + i\boldsymbol{\tau}\cdot\boldsymbol{\pi} = f_\pi \Sigma$ with the constraint $\sigma = \sqrt{f_\pi^2 - \boldsymbol{\pi}^2}$. Σ transforms like $\Sigma \to U_L \Sigma U_R^\dagger$ under $SU(2)_L \times SU(2)_R$ rotations. We are interested in an idealized boost-invariant case [11] where the field $\boldsymbol{\pi} = \boldsymbol{n}(\tau)\pi(\tau, \boldsymbol{x}_\perp)$, i.e. the orientation $\boldsymbol{n}$ of the pion field is only function of the proper time τ defined by $\tau = \sqrt{t^2 - z^2}$ and uniform in spatial rapidity and transverse coordinates $\boldsymbol{x}_\perp$. This would correspond to a single DCC domain in the transverse dimension. The transverse profile of the field amplitude π^2 is determined by the shape and size of the DCC domain, which we shall take a Gaussian form as suggested in numerical simulations [3]

$$g(\boldsymbol{x}_\perp) = \exp[-|\boldsymbol{x}_\perp|^2/R_D^2] \ , \tag{7}$$

where R_D is the size of a DCC domain (typically $R_D = 1 \sim 2$ fm [3]). Such a class of solutions are well known to predict a distinctive neutral pion probability distribution

$$\frac{dP}{df} = \frac{1}{2\sqrt{f}} \ , \tag{8}$$

where P is the probability of find a neutral pion fraction $f = N_{\pi^0}/(N_{\pi^0} + N_{\pi^\pm})$. However, if there exists more than one DCC domain, as it is likely the case especially in heavy-ion collisions where the transverse dimension R_A of the interaction volume is much larger than the size of a DCC domain R_D, the field orientation $\boldsymbol{n}$ could depend on the transverse coordinates $\boldsymbol{x}_\perp$. In this case, the prediction (8) in general no longer holds. Such a possibility

can only be studied in detail in numerical simulations. In this Letter, we shall confine ourselves to the case when the single domain dominates and only comment on how our result might change in a multi-domain case.

According to the "Baked Alaska" scenario [1], the boundary conditions for the solutions are imposed such that the classical waves propagate forward in proper time, starting at an initial time τ_0. In other words, we are interested in a retarded wave that vanishes when $\tau < \tau_0$ and $t < \tau_0$. Such a solution only exists when there is a source located on the hyperbola of equal $\tau = \tau_0$. The "hot" source can be regarded as the summation of the degrees of freedom other than pion modes. Away from the source located near the surface of the light cone, the field propagates freely and one should have the (partial) conservation of (axial) isovector currents

$$\partial_\mu \boldsymbol{V}^\mu = \partial_\mu(\boldsymbol{\pi} \times \partial^\mu \boldsymbol{\pi}) = 0 \ , \qquad (\tau > \tau_0) \tag{9}$$

$$\partial_\mu \boldsymbol{A}^\mu = \partial_\mu(\sigma \partial^\mu \boldsymbol{\pi} - \boldsymbol{\pi} \partial^\mu \sigma) = -m_\pi^2 f_\pi \boldsymbol{\pi} \ . \qquad (\tau > \tau_0) \tag{10}$$

For a function only of τ, a partial derivative $\partial_\mu f(\tau)$ is equal to $(\widetilde{x}_\mu/\tau) df/d\tau$ where $\widetilde{x}_\mu = (t, z, 0, 0)$. Eq.(9) yields the integration

$$\boldsymbol{\pi} \times \partial^\mu \boldsymbol{\pi} = f_\pi^2 \boldsymbol{a} \frac{\widetilde{x}_\mu}{\tau^2} g(\boldsymbol{x}_\perp) \ , \tag{11}$$

where $\boldsymbol{a}$ is a dimensionless constant vector in isospin space. In the chiral limit, the axial current is also conserved, which defines another constant vector $\boldsymbol{b}$ in isospin space with a constraint $\boldsymbol{a} \cdot \boldsymbol{b} = 0$ [5]

$$\boldsymbol{\pi} \partial^\mu \sigma - \sigma \partial_\mu \boldsymbol{\pi} = f_\pi^2 \boldsymbol{b} \frac{\widetilde{x}_\mu}{\tau^2} g(\boldsymbol{x}_\perp) \ . \tag{12}$$

The pion field orientation vector $\boldsymbol{n}(\tau)$ precesses with proper time τ around $\boldsymbol{a}$ in the plane perpendicular to $\boldsymbol{a}$ [6].

The solution (11) exhibits an inverse square singularity as τ goes to zero. As pointed out in [5], the singular oscillation near $\tau = 0$ does not result from the neglect of pion mass: treating the pion mass as a perturbation does not generate qualitative modification

of the solution at small value of τ. Since the ratio $\widetilde{x}_\mu/\tau$ can be regarded as the velocity u_μ of a comoving element with coordinate x_μ, the current diverges outward from the origin indicating the existence of a source near the space-time origin. The boundary condition is satisfied if one multiplies(11) by a step function $\theta(\tau-\tau_0)$. The isovector current in the whole space-time is

$$\boldsymbol{V}_\mu(\tau,\boldsymbol{x}_\perp) = f_\pi^2\boldsymbol{a}\frac{\widetilde{x}_\mu}{\tau^2}\theta(\tau-\tau_0)g(\boldsymbol{x}_\perp)\;, \tag{13}$$

The source term is then calculated (note that $\partial_\mu[\widetilde{x}^\mu f(x)] = (\partial_t,\partial_z,0,0)[\widetilde{x}^\mu f(x)]$)

$$\partial_\mu \boldsymbol{V}^\mu = f_\pi^2\boldsymbol{a}\frac{1}{\tau}\delta(\tau-\tau_0)g(\boldsymbol{x}_\perp)\;. \tag{14}$$

It is clear that the pionic part of isospin (also the electromagnetic) current is not conserved due to the existence of a source term at $\tau=\tau_0$ while the integration constants $\boldsymbol{a}$ and $\boldsymbol{b}$ specify the orientation of the source in isospace or the initial condition. It is also straightforward to calculate the energy density associated with the internal chiral oscillations using the calculated classical currents $\boldsymbol{V}$ and $\boldsymbol{A}$ and the classical solution for the pion field [5,6]

$$\epsilon = \frac{2f_\pi^2(\boldsymbol{a}^2+\boldsymbol{b}^2)}{\tau^2}\;. \tag{15}$$

As required by the chiral symmetry, the energy density depends only on the combination $\boldsymbol{a}^2+\boldsymbol{b}^2$.

The dilepton and photon emissions probe the nature of the source using time-like momenta $q=(q_0,q_{||},\boldsymbol{q}_T)$ with $q^2=M^2$ and $q^2=0$ respectively. The Fourier transformation of the electromagnetic current can be readily calculated in the light cone variables

$$\begin{aligned} J_\mu^{\rm cl}(q) &= f_\pi^2 a_3\int d^4x e^{iqx}\frac{\widetilde{x}_\mu}{\tau^2}\theta(\tau-\tau_0)g(\boldsymbol{x}_\perp) \\ &= -if_\pi^2 a_3\frac{\widetilde{q}_\mu}{M_\perp}\frac{\partial}{\partial M_\perp}\int_{\tau_0}\frac{d\tau}{\tau}\int d\eta e^{iM_\perp\tau\cosh(\eta-y)}\int d^2\boldsymbol{x}_\perp g(\boldsymbol{x}_\perp)e^{-i\boldsymbol{q}_T\cdot\boldsymbol{x}_\perp} \\ &= -\frac{\pi^2R_D^2}{2}f_\pi^2 a_3\frac{\widetilde{q}_\mu}{M_\perp^2}\exp[-q_T^2R_D^2/4][J_0(M_\perp\tau_0)-iN_0(M_\perp\tau_0)]\;, \end{aligned} \tag{16}$$

where $\widetilde{q}_\mu=(q_0,q_{||},0,0)$, $M_\perp=\sqrt{q_0^2-q_{||}^2}$ and J_0 and N_0 are the Bessel functions. For a direct photon, $M_\perp=q_T$. In deriving (16), we have performed the τ-integration from τ_0 to

infinity. Since the electromagnetic current density decreases rapidly with the proper time, the integrated current turns out to be finite and the emission is most effective in the early evolution of the classical field. It is clear from (16) that due to the singular behavior of the Neumann function for small argument, the emission spectra are sensitive to the initial time scale τ_0. The exponential factor $\exp[-q_T^2 R_D^2/4]$ provides a strong q_T cutoff so that the emission is only significant in low momentum region.

The Fourier transformation of the electromagnetic current takes the form $J_\mu^{\rm cl}(q) = \widetilde{q}_\mu f(M_\perp, q_T)$ where f is a scalar function, the orthogonal part is then

$$J_\mu^{\rm tr}(q) = (0, 0, -\boldsymbol{q}_T) f(M_\perp, q_T) \ , \tag{17}$$

and

$$(q^\mu q^\nu - q^2 g^{\mu\nu}) J_\mu^{\rm tr}(q) J_\nu^{\rm tr*}(q) = q_T^2 M_\perp^2 |f(M_\perp, q_T)|^2 \tag{18}$$

$$J^{{\rm tr}\mu}(q) J_\mu^{\rm tr*}(q) = -q_T^2 |f(M_\perp, q_T)|^2 \ . \tag{19}$$

The dilepton and photon distributions are calculated

$$\frac{dN_{\ell^+\ell^-}}{dM_\perp^2 dy dM} = \frac{\alpha^2}{24} (\pi R_D^2)^2 B f_\pi^4 a_3^2 \frac{q_T^2}{M^3 M_T^2} \exp[-q_T^2 R_D^2/2][J_0^2(M_\perp \tau_0) + N_0^2(M_\perp \tau_0)] \ , \tag{20}$$

$$\frac{dN_\gamma}{dy dq_T} = \frac{\alpha}{8} \pi (\pi R_D^2)^2 f_\pi^4 a_3^2 \frac{1}{q_T} \exp[-q_T^2 R_D^2/2][J_0^2(q_T \tau_0) + N_0^2(q_T \tau_0)] \ , \tag{21}$$

where $q_T^2 = M_\perp^2 - M^2$ for the dilepton. Both the dilepton and the photon spectra fall off exponentially for large transverse momentum which is the characteristic of the coherent production from a finite domain. The electromagnetic emission from a DCC domain is thus only important in the low momentum region where the spectra increase as some inverse power law of the momentum.

The constant a_3 can be estimated using the symmetry argument. The important assumption in the disoriented chiral condensate scenario is the equal probability of all internal orientations. According to Eq. (15), the energy density due to the internal chiral oscillations should be equally distributed among all components of vectors $\boldsymbol{a}$ and $\boldsymbol{b}$. One thus has

$$\langle a_i^2 \rangle = \langle b_i^2 \rangle = \frac{1}{6}\frac{\epsilon_0 \tau_0^2}{2 f_\pi^2} \qquad (i = 1, 2, 3) \ . \tag{22}$$

where ϵ_0 is the initial energy density carried by the internal chiral oscillationat at $\tau = \tau_0$, which may be roughly estimated assuming that it is comparable to the thermal energy density (in order for the DCC to have any physical significance). For some typical values $\epsilon_0 \sim 0.5 T_c^4 \sim 0.1$ GeV/fm^3 and $\tau_0 = 5$ fm/c, the average value of a_3^2 is about 4.8.

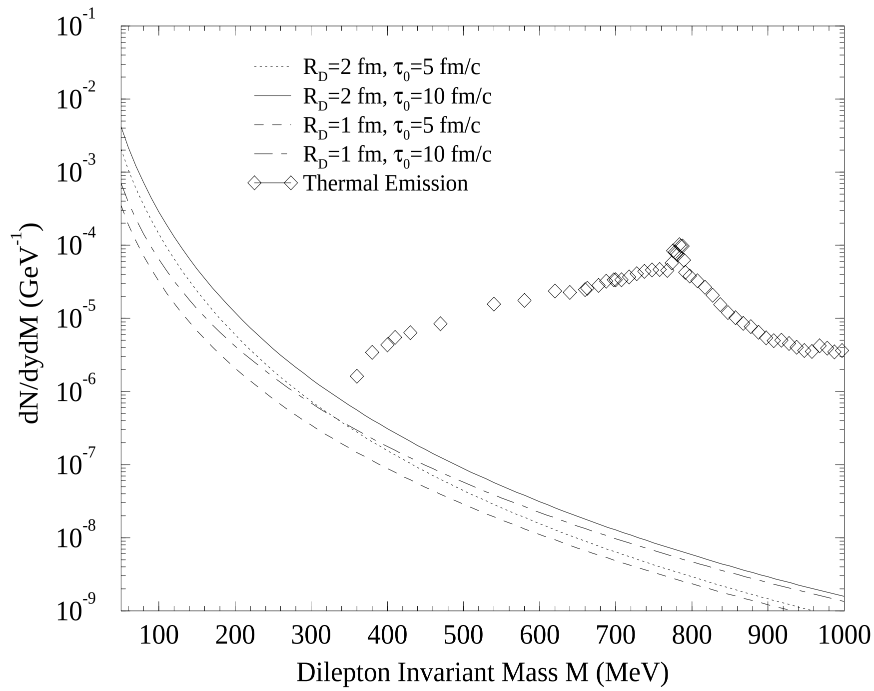

FIG. 1. The dilepton invariant mass spectrum for different choices of initial time scale τ_0 and the coherent field domain size R_D. The initial energy density is assumed to be $\epsilon_0 = 0.1$ GeV/fm^3. The typical thermal spectrum due to the π-π annihilation is also plotted for a comparison

In Fig.1 we numerically integrate (20) over $M_\perp$ and plot the dilepton invariant mass spectrum for different values of $\tau_0 = 5$, 10 fm/c and $R_D = 1, 2$ fm. The a_3 is calculated from (22) assuming that the initial energy density for the internal chiral oscillations is 0.1 GeV/fm^3. Also plotted in Fig.1 is the typical thermal spectrum of dilepton production mainly from π-π annihilations, which is taken from Ref. [10], where we have assumed the initial temperature of 160 MeV, the freeze-out temperature of 130 MeV, and the transverse

size of system $R_A = 5$ fm suitable for heavy-ion collisions. Most notably, there does not exist a pion mass threshold for the dilepton production from the coherent field: the spectrum rises even below $M = 2m_\pi$. The finite pion mass plays no roles in the conserved isovector current whose third component directly couples to the photon. Therefore, the coherent field is most effective in producing lepton pairs in the low mass region $M < 2m_\pi$.

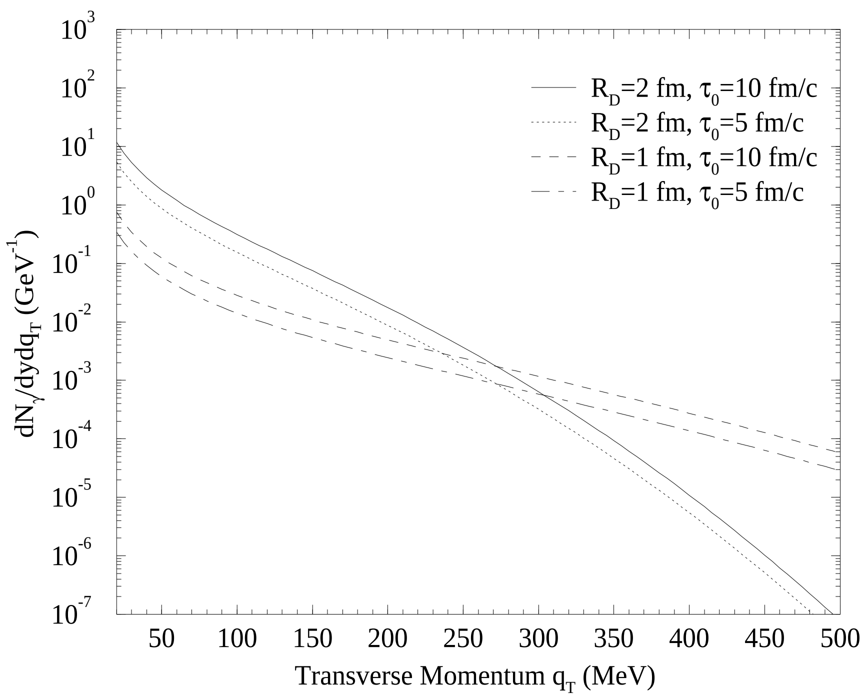

FIG. 2. The direct photon transverse momentum spectrum for different choices of τ_0 and R_D. The initial energy density is assumed to be $\epsilon_0 = 0.1$ GeV/fm^3.

Plotted in Fig.2 is the direct photon transverse momentum spectrum for different choices of τ_0 and R_D. The outstanding feature in the spectrum is the sensitivity on the domain size parameter R_D. The measurement of direct photon spectrum at very small transverse momentum can thus provide a sensitive probe of the spatial extension of the DCC domain.

So far we have only considered a single DCC domain and its orientation oscillation in time. If the multi-domain configurations dominate the DCC production, the dilepton and photon can be coherently produced from different oriented regions. In particular, if there are two domains with opposite orientation (the molecule type), the singular rise of the spectra

near zero momentum will be suppressed and spectra at the moderate momentum will be enhanced. We plan to address these interesting questions in future publications.

In conclusion, we have suggested that the study of electromagnetic emission can provide some information on the space-time history of a disoriented chiral condensate. The emissions due to the coherent isospin oscillation can be an important additional source to the electromagnetic signals from a dense hadronic matter, especially in low mass (momentum) region. It would be very interesting to see whether or not the qualitative features suggested in this paper can survive in more realistic models, and indeed may provide some mechanism for the observed low mass dilepton enhancement by the CERES and HELIOS collaborations at CERN SPS experiments [12].

We would like to thank Hans Thomas Elze and Ina Sarcevic for useful discussions. This work was supported through the U.S. Department of Energy under Contracts Nos. DE-FG03-93ER40792 and DE-AC03-76SF00098.

REFERENCES

[1] J.D. Bjorken, K.L. Kowalski, C.C. Taylor, "Baked Alaska", SLAC-PUB-6109, 7th Les Rencontres de Physique de la Vallee d'Aoste: Results and Perspectives in Particle Physics, La Thuile, Italy, 7-13 March, 1993; A. A. Anselm and M. G. Ryskin, Phys. Lett. B **266**, 482 (1991); K. Rajagopal and F. Wilczek, Nucl. Phys. **B404**, 577 (1993).

[2] S. Gavin and B. Müller, Phys. Lett. **B329**, 486 (1994).

[3] S. Gavin, A. Gocksch, and R. D. Pisarski, Phys. Rev. Lett. **72**, 2143 (1994);M. Asakawa, Z. Huang, and X.-N. Wang, Phys. Rev. Lett. **74**, 3126 (1995);F. Cooper, Y. Kluger, E. Mottola, and J. P. Paz, Phys. Rev.**D51**, 2377 (1995); J. Randrup, LBL Preprint LBL-38125 (1996).

[4] E. Shuryak, Phys. Lett. **B79**, 135 (1978); K. Kajantie and H.I. Miettenen, Z. Phys. **C9**, 341(1981); K. Kajantiem J. Kapusta, L. McLerran and A. Mekjian, Phys. Rev. **D34**, 2476 (1986).

[5] J. -P. Blaizot and A. Krzywcki, Phys. Rev. D **46**, 246 (1992).

[6] Z. Huang and M. Suzuki, Phys. Rev. **D53**, 891 (1996).

[7] L. McLerran and T. Toimela, Phys. Rev. **D31**, 545(1985).

[8] P.V. Ruuskanen, in *Particle Production in Highly Excited Matter*, edited by H. Gutbrod (Plenum, New York, 1993).

[9] H. Weldon, Phys. Rev. **D42**, 2384 (1990).

[10] Z. Huang, Phys. Lett. **B361**, 131 (1995).

[11] J. Bjorken, Phys. Rev. **D27**, 140 (1983).

[12] G. Agakichiev, *et al.*, Phys. Rev. Lett. 75, 1272 (1995).